Predicting Reservoir Behaviour Using Mathematical Models 2: *Water Underrunning*

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Introduction

Forecasting the behaviour of a reservoir is one of the more important but complicated task of engineers in the oil industry. Knowledge of reserves remaining in a reservoir is vital to planning optimum depletion of a field. Unfortunately, the engineer assigned the task of Forecasting the behavior of a reservoir is one of the most important but complicated talks of engineers in the oil and gas industry. Knowledge of reserves remaining in a reservoir is vital to planning optimum depletion of a field. Unfortunately, the engineer assigned the task of predicting reserves often face a difficult choice. For the most accurate answer, he can use a computer program that takes into account all the pertinent factors, but this approach is usually expensive and time consuming, and requires and time consuming, and requires a detailed knowledge of the reservoir. On the other hand can use conventional one-dimensional displacement calculations that are easily applied but that in some cases do not adequately describe the reservoir flow system. The purpose of this paper is to describe a middle-ground approach that is in special situations has many of the advantages of the above methods without their more serious drawbacks. This approach uses mathematical model that describe the principle flow mechanisms and can be quickly applied by hand calculations.

Keywords: *Models, Depletion, Reservoir, Predict, Reserves, Mobility Ratio.*

Physical Description of Problem

When water displaces viscous oils, conditions are seldom favourable for uniform contacting on the sand by water0producing rates, because of the demands of economics, must usually exceed critical rates, making it impossible for gravitational forces to maintain a stable displacement front. The mobility ratio is slightly unfavourable in these reservoirs and water tends to channel and by-pass oil.

Since water is denser than oil, it seeks the bottom of the interval and channels or “tongues” under the oil. When water arrives in the region under a producing well, it tends to cone up into the well, if vertical permeability exists, and be produced with the oil as shown in fig 3. Production histories in such reservoirs are characterized by early breakthrough of water into all of the producing walls, followed by an extended period of gradually increasing water oil ratios.

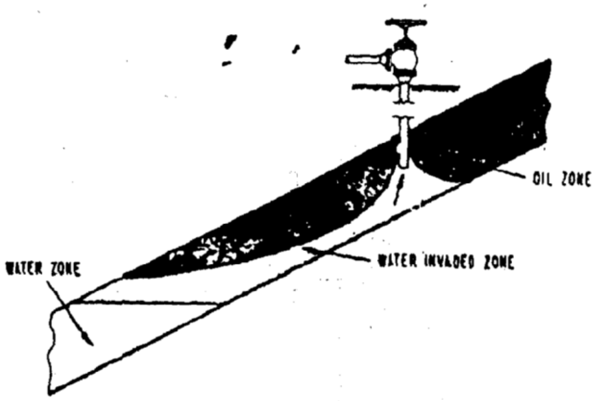


Fig. 3—Mechanics of water underrunning and coning.

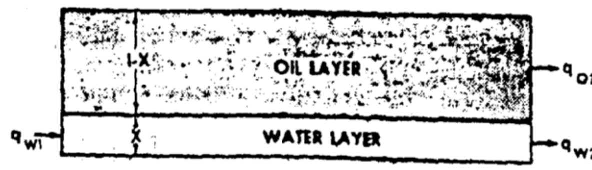


Fig. 4—Model for water underrunning.

The Mathematical Model

Consider a cross-section of sand contained viscous oil as shown in fig 4. Assume that water underruns the oil because it is denser than oil, and assume that the water layer is of constant thickness. At a particular time, the fraction of the total thickness occupied by water is x and the fraction occupied by oil is 1-x. oil and water flow horizontally with the viscous pressure generated by the invading water. It is assumed that oil is produced, the water layer moves vertically upward with no resistance.
The average oil flow rate along the length is one-half the oil production since no oil enters the inflow face and the average water flow is the rate of water production plus one-half the rate of oil production. If we assume that both water and oil floe horizontally through unit width, we obtain;

$$\Delta P_w = \frac{\left(q_w + \frac{q_o}{2}\right)\mu_w L}{k_{w,\ ro} x} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

And

$$\Delta P_o = \frac{\left(\frac{q_o}{2}\right)\mu_o L}{k_o(1-x)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

In which $k_{w,\ ro}$ is the effective permeability to water at residual saturation.
Produced oil is replaced by water moving vertically upward, so

$$q_o = \frac{dx}{dt} L\phi(1 - S_{wi} - S_{or}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

By assuming that the pressure drop is the same through the oil and water layers, equating (1) to (2). Solving for q_o, eliminating q_o, with equation (3) and grouping variable we obtain

$$\frac{q_w dt}{Wp(1 - S_{wi} S_{or}} = \frac{k_{w,\ ro,}\mu_o}{2k_o\mu_w} \quad \frac{x\ d\ d}{1-x} - \frac{dx}{2} \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Integrating equation (4), we obtain;

$$W_p = \frac{q_u t}{L\phi(1 - S_{wi} - S_{oi}} = \frac{k_{w,\ ro}\mu_o}{2k_o\mu_w}\left[-x - In(1-x) - \frac{x}{2} \ldots\ldots\ldots\ldots\ldots\ldots..(5)\right.$$

In which W_p is the total water production in displaceable pore volumes. Water influx is found by $W_p = W_p + x$.

Cumulative oil recovery, Np, in displacement pore volumes is equal to x. after breakthrough, the instantaneous produced water-oil ratio is given by;

$$\frac{q_w}{q_o} = \frac{k_{w,\ ro}\mu_o}{2k_o\mu_w}\left(\frac{x}{1-x}\right) - \frac{1}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(6)$$

Given;

S_{wi} = 0.11

S_{or} = 0.20

μ_o = 50cp

μ_w = 0.4cp

k_o = 1.0 darcy

$k_{w,\ ro}$ = 0.25 darcy

Substituting the above total into equation (5) provides the following;

$$W_e = \frac{(50)(0.25)}{(2)(0.4)(1)}[-x - In(1-x) + \frac{x}{2} = -15.7\, In\,(1-x) - 15.2x$$

A table may now be prepared relating x, the fraction of the thickness invaded (or recovery in percent of displaceable oil), to water influx and produces water-oil ratio (table 4)

Table 4 – PREDICTED BEHAVIOR WITH WATER UNDERRUNNING

Oil Recovery (displaceable volume or fraction swept)	Water influx (displaceable volumes)	Water oil ratio	Water cut (percent)
0.1	0.13	1.24	55.2
0.2	0.45	3.42	77.4
0.3	1.04	6.22	86.1
0.4	1.92	10.00	90.9
0.5	3.26	15.00	93.7
0.6	5.28	23.00	95.8

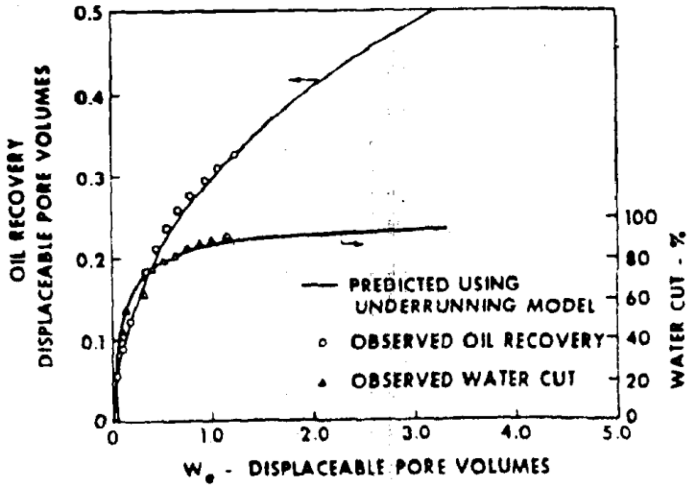
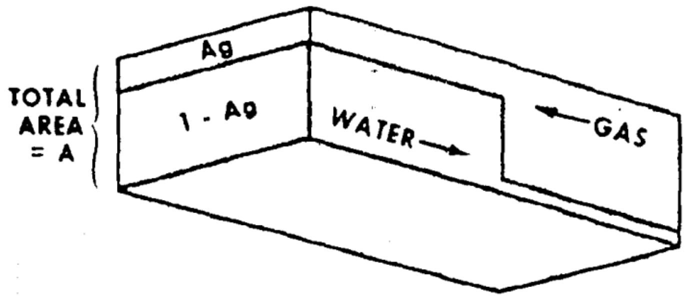


Fig. 5—Comparison of predicted and actual reservoir behavior.



Model for gravity segregation of water.

The model predicts that water break through will have occurred in all wells when 5 percent of the displaceable oil is produced and that water cuts will exceed 90 percent before 40 percent of the displacement oil is produced.

Comparison with Field Data

Results obtained by the above calculations are compared in fig 5 with recoveries and water cuts observed in the field as fractions of water influx. Note that the field data are closely matched by the curves derived by the simple mathematical model. Calculations were also made of conditions existing in the field. Recovery at water breakthrough was predicted to be 40 percent, which compares well with a value of 42 percent calculated by dividing the oil production by the volume originally in place in the water invaded regions.

Conclusion

Thus, it is concluded that this model can be used to predict future production characteristics in different fields. It is also concluded that where assumptions is used in deriving the model apply, it may have utility in predicting behavior in other reservoirs it can be demonstrated that it will match past history. While this model predicts that water will channel and bypass oil, it assumes complete areal heterogeneities, only part of the reservoir area may be contacted by water and adjustments or conformance factors will be needed to match reservoir behaviour with the model.

Nomenclature

S = saturation
t = time
u = flow rate per unit area
W = volume of water
x = fraction of the thickness
Z = vertical distance moved by a saturation
$\propto$ = angle of dip
μ = viscosity
P = density
ϕ = porosity

Subscripts

e = influx (or "effective" in coning)
g = gas
i = initial or injection
o = oil
r = relative or residual
T = total
V = porosity

References

Levereti, M.C (2008); "Capilary Behavior in porous solids" Trans C, ATME 142, 152 – 159

Muskat, M (2007); The flow orf Compressible fluids through porous media Trans, ATME

Joslin, W.J (2002); "Applying the Frontal Advance Equation to vertical Segregation Reservoirs" Journal of Petroleum Technology pg. 87 – 94

Matthews, C.S and Lefkovits, H.C (2005); "Gravity Drainage performance of Depletion type Reservoirs in the stripper stage". "Trans., AIME 207, 265 – 274

Hail, H.N (2003); "Predcting Gravity Drainage Performance Using three Dimensional model" I. Pet. Tech. page 517 - 524